CHAPTER 12 -- WAVE MOTION

12.1) The relationship between a wave's *frequency* ν, its *wavelength* λ, and its *wave velocity* v is $v = \lambda\ \nu$. For sound in air, the wave velocity is approximately $v = 330\ m/s$. To get the wavelength:

a.) For $\nu = 20\ hz$:

$$\begin{aligned}\lambda &= v/\nu \\ &= (330\ \text{m/s})/(20\ \text{hz}) \\ &= 16.5\ \text{meters} \qquad \text{(around 50 feet).}\end{aligned}$$

Note: Technically, the units of *frequency* are *seconds*$^{-1}$ and of *wavelength* are *meters*. The *cycles* term in the frequency units is a label, being the same for MKS, CGS, and the English system of units. This means that by dividing frequency into velocity we get the units $(m/s)/(1/s) = meters$. If you had included the cycles label, that division would have yielded units of $(m/s)/(cycles/s) = meters/cycle$. There is really nothing wrong with this--it is a literal description of what the wavelength is (the number of meters there is in one wave--one cycle), but using it could potentially get you in trouble later. Best go with *seconds*$^{-1}$ for simplicity.

b.) For $\nu = 20{,}000\ hz$:

$$\begin{aligned}\lambda &= v/\nu \\ &= (330\ \text{m/s})/(20{,}000\ \text{hz}) \\ &= .0165\ \text{meters} \quad \text{(a little over half an inch).}\end{aligned}$$

12.2)

a.) The sketch is shown on the next page.

b.) If the first wave has an amplitude of $A_1 = 1\ meter$, the second largest amplitude wave will have an amplitude of approximately $A_2 = .33\ meters$ and the third approximately $A_3 = .2\ meters$.

Note that the largest wave (the *first wave* as defined above) has one half-wavelength in the same space that the *second wave* has 3 half-wavelengths and the *third wave* has 5 half-wavelengths. That means that if the first wave has a frequency of $\nu_1 = 1\nu$, the second wave will have a frequency of $\nu_2 = 3\nu$ and the third wave a frequency of $\nu_3 = 5\nu$.

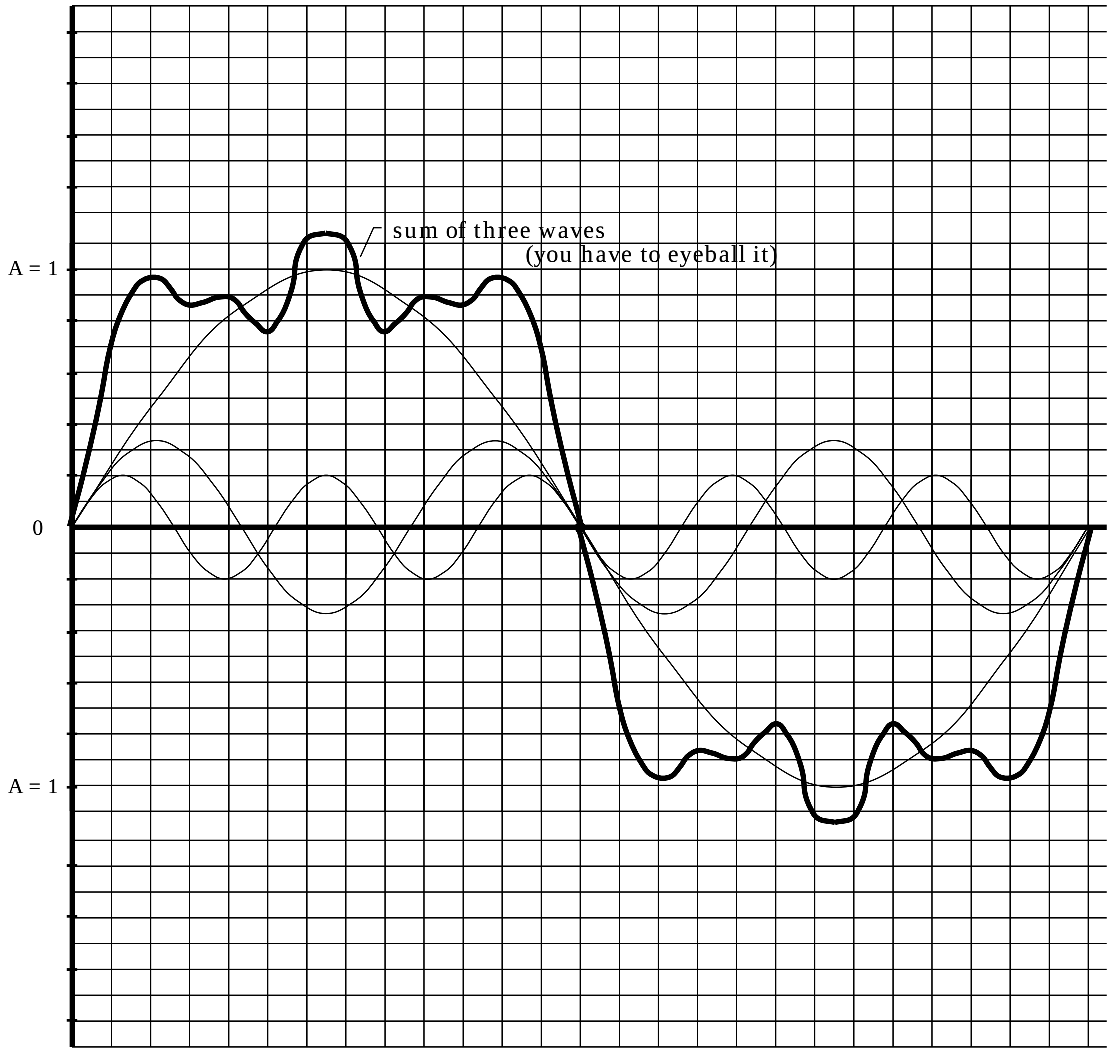


FIGURE I

Frequency is proportional to the *angular frequency*. That means that if the *angular frequency* of the first wave is $\omega_1 = 1\ rad/sec$, the second wave's angular frequency will be $\omega_2 = 3\ rad/sec$ and the third wave's angular frequency will be $\omega_3 = 5\ rad/sec$.

Putting it all together, remembering that the general algebraic expression for a sine wave with no phase shift is *A sin* ωt, we get:

$$
\begin{aligned}
y_{tot} &= A_1 \sin \omega_1 t + A_2 \sin \omega_2 t + A_3 \sin \omega_3 t \\
&= 1 \sin t + .33 \sin 3t + .2 \sin 5t. \\
&= (1/1) \sin t + (1/3) \sin 3t + (1/5) \sin 5t.
\end{aligned}
$$

c.) The first six terms of the series are:

$$y_t = 1\sin 1t + (1/3)\sin 3t + (1/5)\sin 5t + (1/7)\sin 7t + (1/9)\sin 9t + (1/11)\sin 11t.$$

d.) The waveform is shown to the right. It is called a *square wave*.

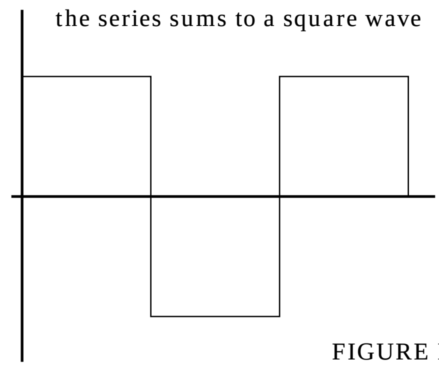


FIGURE II

12.3)

a.) At *t = 0*:

$$
\begin{aligned}
y &= 12 \sin (25x - .67(0)) \\
&= 12 \sin 25x.
\end{aligned}
$$

This function is graphed in Figure III to the right.

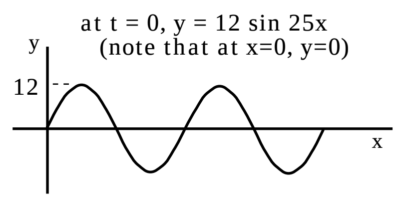


FIGURE III

At *t = 1 second*:

$$
\begin{aligned}
y &= 12 \sin (25x - .67(1)) \\
&= 12 \sin (25x - .67).
\end{aligned}
$$

This function is graphed in Figure IV to the right.

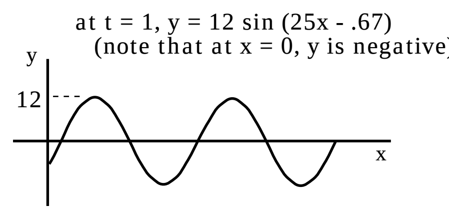


FIGURE IV

b.) The wave is moving to the right (note that its peaks are further to the right at *t = 1 second* than they are at *t = 0 seconds*).

c.) *Positive* values for the time dependent part of this equation yield wave motion (in time) to the left (in the negative *x direction*); *negative* values for the time dependent part of this equation yield wave motion to the right (i.e., in the *+x direction*), as was pointed out in *Part b*.

d.) The *angular frequency* is .67 rad/sec. As:

$$
\begin{aligned}
\omega &= 2\pi\nu \\
\Rightarrow \quad \nu &= \omega/2\pi \\
&= (.67 \text{ rad/sec})/2\pi \\
&= .107 \text{ Hz.}
\end{aligned}
$$

e.) The period is:

$$
\begin{aligned}
T &= 1/\nu \\
&= 1/(.107 \text{ Hz}) \\
&= 9.35 \text{ sec/cycle.}
\end{aligned}
$$

f.) We know that the wave number is $k = 25\ m^{-1}$. The wavelength is related to the wave number by:

$$
\begin{aligned}
k &= 2\pi/\lambda \\
\Rightarrow \quad \lambda &= 2\pi/k \\
&= 2\pi/(25 \text{ m}^{-1}) \\
&= .25 \text{ meters.}
\end{aligned}
$$

Note: Just as *angular frequency* tells you how many radians the wave sweeps through per unit time at a given point, the *wave number* tells you how many radians of wave there are per meter of wave.

Look at the units if this isn't clear. The equation states that there are (*2π radians/wavelength*) divided by (λ *meters* of wave per wavelength), or $2\pi/\lambda$ *radians per meter* of wave. Put another way, if $k = 2\pi\ rad/m$, we are being told that one full cycle of wave (i.e., 2π radians worth) spans *one meter*.

g.) Wave velocity:

$$
\begin{aligned}
v &= \lambda\nu. \\
&= (.25 \text{ m})(.107 \text{ Hz}) \\
&= .02675 \text{ m/s.}
\end{aligned}
$$

h.) The amplitude is 12 meters (by inspection).

12.4) We know that the frequency is 225 Hz, the amplitude is .7 meters, and the wave velocity is 140 m/s. Knowing the wave velocity, we can write:

$$
\begin{aligned}
v &= \lambda \nu \\
\Rightarrow \quad \lambda &= v/\nu \\
&= (140 \text{ m/s})/(225 \text{ Hz}) \\
&= .622 \text{ meters.}
\end{aligned}
$$

Traveling waves have a general algebraic expression of:

$$
\begin{aligned}
y &= A \sin (kx + \omega t) \\
&= A \sin [(2\pi/\lambda)x + 2\pi \nu t] \\
&= .7 \sin \big[[2\pi/(.622 \text{ m})]x + 2\pi(225 \text{ Hz})t\big] \\
&= .7 \sin (10.1x + 1413.7t).
\end{aligned}
$$

12.5) Dividing out the coefficient of the α term to get this equation in the right form (i.e., the standard *simple harmonic motion* equation), we get:

$$\alpha + (3g/2L)\,\theta = 0.$$

a.) The angular frequency for this motion is:

$$
\begin{aligned}
\omega &= (3g/2L)^{1/2} \\
&= [3(9.8 \text{ m/s}^2)/2(.8 \text{ m})]^{1/2} \\
&= 4.29 \text{ rad/sec.}
\end{aligned}
$$

Knowing this, we can find the natural frequency-of-oscillation for this system:

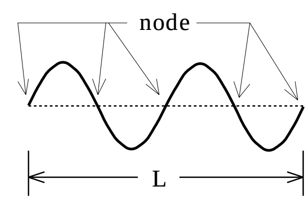
$$
\begin{aligned}
\nu &= \omega/2\pi \\
&= (4.29 \text{ rad/sec})/2\pi \\
&= .683 \text{ Hz.}
\end{aligned}
$$

b.) The period is:

$$
\begin{aligned}
T &= 1/\nu \\
&= 1/(.683 \text{ Hz}) \\
&= 1.46 \text{ sec/cycle.}
\end{aligned}
$$

If the frequency (hence period) of the applied force is close to the natural frequency (hence period) of the system, resonance will occur and the amplitude of the motion will grow immensely. If not, the applied force will fight the natural motion and the net effect will be small amplitude motion. The period in *Part i* (1.31 seconds) fits into the latter category; the period in *Part ii* (1.47 seconds) fits into the former category.

12.6) We know $\nu = 800\ Hz;\ L = .3\ meters$; and there are 5 nodes with one at each end (that is, the string is split into four sections by the three remaining nodes). A sketch of the system is shown to the right.



To get the velocity, we will use $v = \lambda\ \nu$. We know ν; we need λ. To get it, notice that there are TWO full wavelengths in the length L. Mathematically:

$$\begin{aligned} 2\lambda &= L \\ &= .3\ m \\ \Rightarrow \quad \lambda &= .15\ m. \end{aligned}$$

Putting it all together, we get:

$$\begin{aligned} v &= \lambda\ \nu \\ &= (.15\ m)(800\ Hz) \\ &= 120\ m/s. \end{aligned}$$

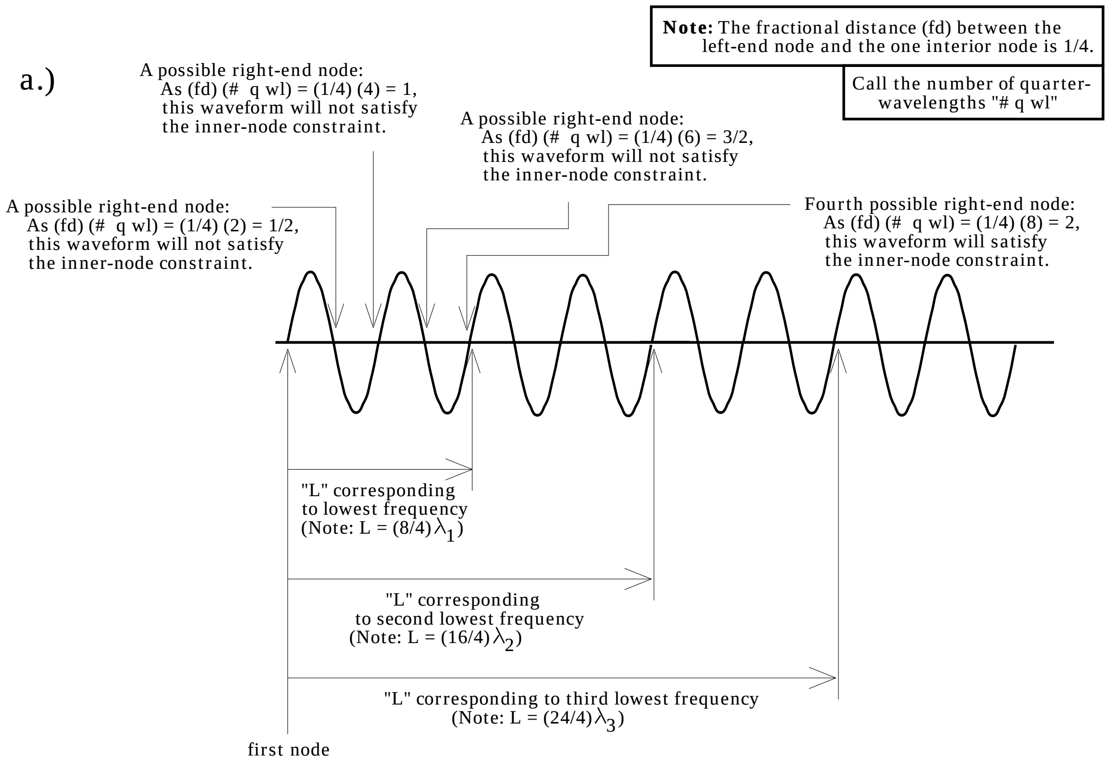
12.7) Calculations for all parts follow the sketches on the next few pages.

a.) We need a node at both ends and one *L/4* units from the left end. The *sine wave* on the next page depicts the various possibilities.

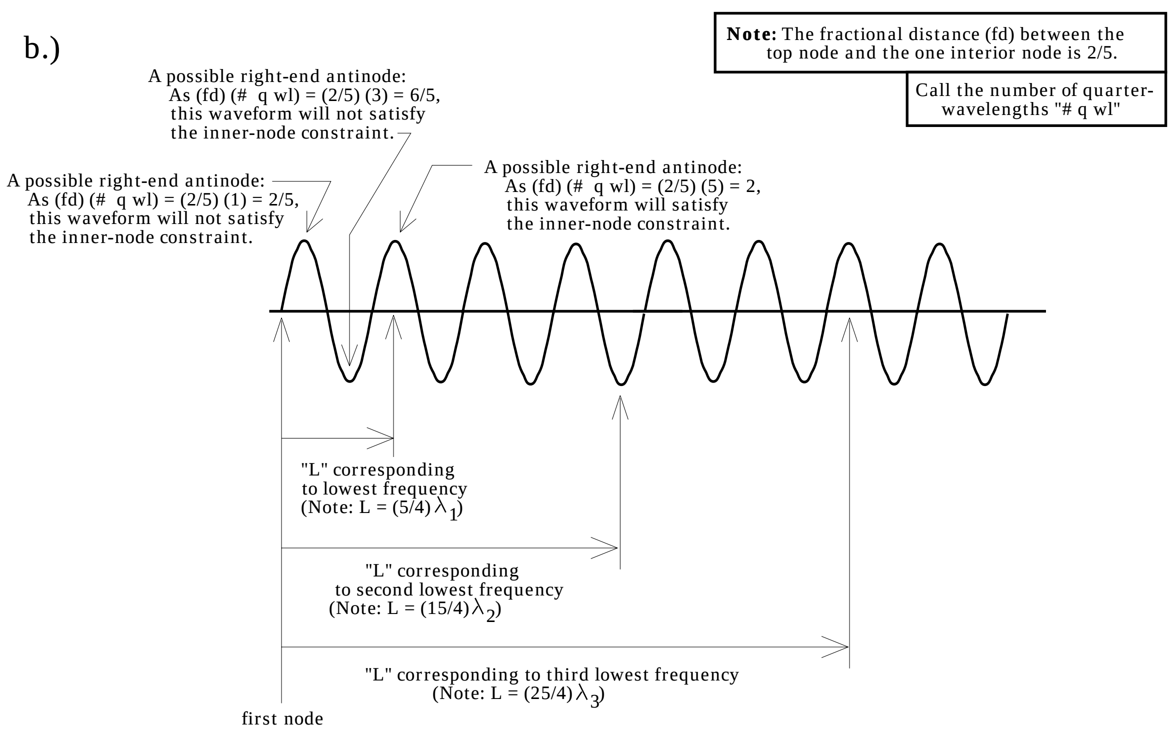
b.) We need a node at the ceiling, an antinode at the free end, and a node *(2/5)L* of the way down from the ceiling. The *sine wave* on the next page depicts the various possibilities.

c.) We need an antinode at the top, a node at the bottom, and a node at *L/3* from the top. The *sine waves* on the next two pages depict the various possibilities (I've pictured the sine wave horizontally for simplicity).

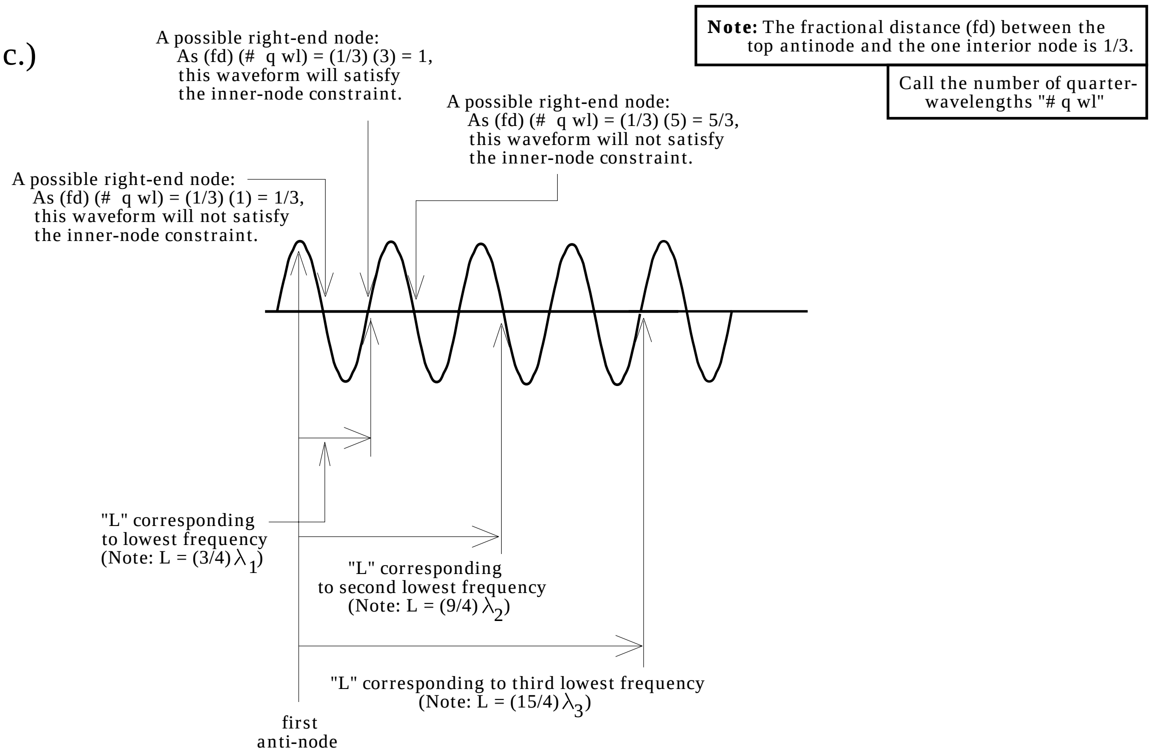
a.)


Note: The fractional distance (fd) between the left-end node and the one interior node is 1/4.
Call the number of quarter-wavelengths "# q wl"
A possible right-end node: As (fd) (# q wl) = (1/4) (4) = 1, this waveform will not satisfy the inner-node constraint.
A possible right-end node: As (fd) (# q wl) = (1/4) (6) = 3/2, this waveform will not satisfy the inner-node constraint.
A possible right-end node: As (fd) (# q wl) = (1/4) (2) = 1/2, this waveform will not satisfy the inner-node constraint.
Fourth possible right-end node: As (fd) (# q wl) = (1/4) (8) = 2, this waveform will satisfy the inner-node constraint.
"L" corresponding to lowest frequency (Note: L = (8/4) λ1)
"L" corresponding to second lowest frequency (Note: L = (16/4) λ2)
"L" corresponding to third lowest frequency (Note: L = (24/4) λ3)
first node

b.)


Note: The fractional distance (fd) between the top node and the one interior node is 2/5.
Call the number of quarter-wavelengths "# q wl"
A possible right-end antinode: As (fd) (# q wl) = (2/5) (3) = 6/5, this waveform will not satisfy the inner-node constraint.
A possible right-end antinode: As (fd) (# q wl) = (2/5) (1) = 2/5, this waveform will not satisfy the inner-node constraint.
A possible right-end antinode: As (fd) (# q wl) = (2/5) (5) = 2, this waveform will satisfy the inner-node constraint.
"L" corresponding to lowest frequency (Note: L = (5/4) λ1)
"L" corresponding to second lowest frequency (Note: L = (15/4) λ2)
"L" corresponding to third lowest frequency (Note: L = (25/4) λ3)
first node

c.)



AS FOR THE NUMBERS:

a.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/beam length ratio that leaves:

$$L = 6\lambda_3$$
$$\Rightarrow \quad \lambda_3 = L/6.$$

Using this with $v_{beam} = \lambda\, \nu$, we get:

$$\begin{aligned}\nu_3 &= v_{beam}/\lambda_3 \\ &= v_{beam}/(L/6) \\ &= 6v_{beam}/L.\end{aligned}$$

b.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/string length ratio that leaves:

$$L = (25/4)\lambda_3$$
$$\Rightarrow \quad \lambda_3 = 4L/25.$$

Using this with $v_{str} = \lambda\, \nu$, we get:

$$\begin{aligned}\nu_3 &= v_{str}/\lambda_3 \\ &= v_{str}/(4L/25) \\ &= 25v_{str}/4L.\end{aligned}$$

c.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/air-column-length ratio that leaves:

$$L = (15/4)\lambda_3$$
$$\Rightarrow \quad \lambda_3 = 4L/15.$$

Using this with $v_{air} = \lambda\, \nu$, we get:

$$\begin{aligned}\nu_3 &= v_{air}/\lambda_3 \\ &= v_{air}/(4L/15) \\ &= 15v_{air}/4L.\end{aligned}$$

We can go a little further with this problem because we know that the velocity of sound in air is approximately 330 m/s. Putting that in yields:

$$\begin{aligned}\nu_3 &= 15v_{air}/4L \\ &= 15(330\text{ m/s})/4L \\ &= 1237.5/L.\end{aligned}$$

NOTE: The hard part of these problems is finding the appropriate *piece of sine-wave* (relating its wavelength to *L* isn't hard at all). Make sure you understand how to do this. If you are confused, come see me!